1. Homework 4

Due: In Lecture 9-28

Problem 1. Show that if M is a k-dimensional differentiable manifold with boundary in $\mathbb{R}^n$, then ∂M is a $(k-1)$-dimensional manifold (without boundary) in $\mathbb{R}^n$.

Problem 2. Identify the set of real 2×2 matrices with R^4.

(a) Show that the subset M of matrices of rank 1 is a 3-dimensional differentiable manifold in R^4.

(b) Show that the set of 2×2 matrices of determinant 1 is a 3-dimensional differentiable submanifold of R^4.

Problem 3 Show that M_x consists of the tangent vectors to smooth curves in M passing through x.

Problem 4 (a) Find a basis for the tangent space to the unit 2-sphere S^2 in R^3 at the point $p = (a, b, c)$.

(b) What is the tangent space to the hyperboloid in $\mathbb{R}^3$ defined by the equation $x^2 + y^2 - z^2 = a^2$ at the point $(a, 0, 0)$?

Problem 5 The orthogonal group $O(n)$ consists of all $n \times n$ matrices A such that $AA^T = I$. Identify the set $M(n)$ of all $n \times n$ matrices with Euclidean space of dimension n^2, and then show that the orthogonal group $O(n)$ is a differentiable submanifold. What is its dimension?